[2] $f(t) = \boxed{\begin{cases} -10t+16, & \text{IF } t<2 \\ -5t-2, & \text{IF } t\geq 2 \end{cases}}$ ④ $= -10t+16+u(t-2)(-5t-2+10t-16)$

$= \boxed{-10t+16+u(t-2)(5t-18)}$ ④

$\mathcal{L}\{f(t)\} = \boxed{-\frac{10}{s^2}+\frac{16}{s}}$ ② $+ e^{-2s}\mathcal{L}\{5(t+2)-18\}$

$= -\frac{10}{s^2}+\frac{16}{s} + e^{-2s}\mathcal{L}\{\boxed{5t-8}\}$ ②

$= -\frac{10}{s^2}+\frac{16}{s} + \boxed{e^{-2s}\left(\frac{5}{s^2}-\frac{8}{s}\right)}$ ②

$= \frac{16s-10}{s^2} + e^{-2s}\frac{5-8s}{s^2} = -2\left(\frac{5-8s}{s^2}\right)+e^{-2s}\left(\frac{5-8s}{s^2}\right)$

$= F(s)$

④ $\boxed{\begin{aligned} &s^2\mathcal{L}\{y\} - s\overset{-3}{\cancel{y(0)}} - \overset{1}{\cancel{y'(0)}} \\ &+2(s\mathcal{L}\{y\} \qquad - \overset{-3}{\cancel{y(0)}}) \\ &+5\mathcal{L}\{y\} \end{aligned}} = F(s)$

$(s^2+2s+5)\mathcal{L}\{y\} + 3s+5 = F(s)$

$\mathcal{L}\{y\} =$ ② $\boxed{-\frac{3s+5}{(s+1)^2+4}}\ \boxed{-2\cdot\frac{5-8s}{s^2((s+1)^2+4)} + e^{-2s}\frac{5-8s}{s^2((s+1)^2+4)}}$ ②

$=$ ④ $\boxed{-\frac{3(s+1)+1(2)}{(s+1)^2+4}} - 2\left(\boxed{-\frac{2}{s}+\frac{1}{s^2}+\frac{2(s+1)+\frac{1}{2}(2)}{(s+1)^2+4}}\right)$ FOR WORK ⑳

$+ e^{-2s}\left(-\frac{2}{s}+\frac{1}{s^2}+\frac{2(s+1)+\frac{1}{2}(2)}{(s+1)^2+4}\right)$

④

$y = \boxed{-3e^{-t}\cos 2t - e^{-t}\sin 2t}$ ⑥

$-2\left(\boxed{-2+t+2e^{-t}\cos 2t+\frac{1}{2}e^{-t}\sin 2t}\right)$

⑧ $+\boxed{u(t-2)\left(-2+(t-2)+2e^{-(t-2)}\cos 2(t-2)+\frac{1}{2}e^{-(t-2)}\sin 2(t-2)\right)}$

$=$ ⑩ $\boxed{\begin{aligned} &4-2t-7e^{-t}\cos 2t-2e^{-t}\sin 2t \\ &+u(t-2)\left(t-4+2e^{2-t}\cos(2t-4)+\frac{1}{2}e^{2-t}\sin(2t-4)\right) \end{aligned}}$

$$\frac{3s+5}{(s+1)^2+4} = \frac{\cancelto{3}{A}(s+1)+\cancelto{1}{B}(2)}{(s+1)^2+4} \longrightarrow 3s+5 = A(s+1)+B(2)$$

$$s=-1: \quad 2 = 2B \longrightarrow B = 1$$

COEF OF s: $\quad 3 = A$

$$\frac{5-8s}{s^2((s+1)^2+4)} = \boxed{\frac{\cancelto{-2}{A}}{s} + \frac{\cancelto{1}{B}}{s^2} + \frac{\cancelto{2}{C}(s+1)+\cancelto{\frac{1}{2}}{D}(2)}{(s+1)^2+4}} \quad 4$$

4

$$\boxed{5-8s = As((s+1)^2+4) + B((s+1)^2+4) + C(s+1)s^2 + D(2s^2)}$$

$s=0$: $\quad 5 = B(5) \longrightarrow B = 1$

COEF OF s: $\quad -8 = 5A+2B \longrightarrow A = \frac{1}{5}(-8-2B) = -2$

$s=-1$: $\quad 13 = A(-1)(4) + B(4) + D(2) \longrightarrow D = \frac{1}{2}(13+4A-4B)$

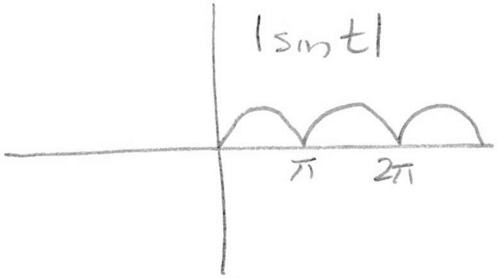
$$= \frac{1}{2}$$

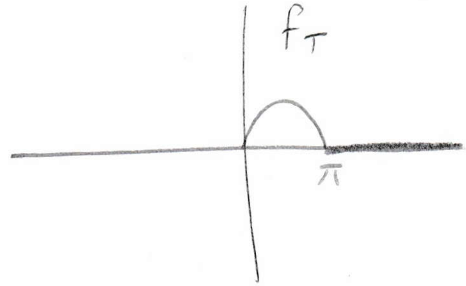
COEF OF s^3: $\quad 0 = A + C \longrightarrow C = -A = 2$

SANITY CHECK: $s = -2$

$$\boxed{\begin{aligned} &\text{LHS} \quad \frac{5+16}{4(5)} = \frac{21}{20} = 1\tfrac{1}{20} \\ &\text{RHS} \quad \frac{-2}{-2} + \frac{1}{4} + \frac{2(-1)+\frac{1}{2}(2)}{5} \\ &\quad = 1 + \frac{1}{4} - \frac{1}{5} \\ &\quad = 1 + \frac{5-4}{20} \\ &\quad = 1\tfrac{1}{20} \end{aligned}} \quad 4$$

[3]





4 $$\boxed{s^2\mathcal{L}\{y\} - s\,\cancel{y(0)}^{\,6} - \cancel{y'(0)}^{\,-5} - \mathcal{L}\{y\}} = F(s)$$

$$(s^2-1)\mathcal{L}\{y\} - 6s + 5 = F(s)$$

$$\mathcal{L}\{y\} = \boxed{\frac{6s-5}{(s-1)(s+1)}}$$ 2

$$+ \boxed{\frac{1+e^{-\pi s}}{1-e^{-\pi s}} \; \frac{1}{(s^2+1)(s+1)(s-1)}}$$ 2

6 $$= \boxed{\frac{\frac{1}{2}}{s-1} + \frac{\frac{11}{2}}{s+1}}$$ FOR WORK

6 $$+ \boxed{\left(1 + \sum_{n=1}^{\infty} 2e^{-n\pi s}\right)} \cdot \boxed{\left(\frac{-\frac{1}{2}}{s^2+1} - \frac{\frac{1}{4}}{s+1} + \frac{\frac{1}{4}}{s-1}\right)}$$

FOR WORK 16

$$y = \boxed{\frac{1}{2}e^{t} + \frac{11}{2}e^{-t}}$$ 2

$$\boxed{-\frac{1}{2}\sin t + \frac{1}{4}e^{t} - \frac{1}{4}e^{-t}}$$ 3

8 $$\boxed{+\sum_{n=1}^{\infty} 2u(t-n\pi)\left(-\frac{1}{2}\underbrace{\sin(t-n\pi)}_{\substack{=\sin t,\ \text{IF } n \text{ EVEN} \\ =-\sin t,\ \text{IF } n \text{ ODD}}} + \frac{1}{4}e^{t-n\pi} - \frac{1}{4}e^{n\pi-t}\right)}$$

$$f_T(t) = \boxed{\begin{cases} \sin t, & \text{IF } t \in [0,\pi) \\ 0, & \text{IF } t \geq \pi \end{cases}}$$ 4

$$= \sin t + u(t-\pi)(0 - \sin t)$$

4 $$= \boxed{\sin t - u(t-\pi)\sin t}$$

$$\mathcal{L}\{f_T\} = \frac{1}{s^2+1} - e^{-\pi s}\mathcal{L}\{\sin(t+\pi)\}$$

2 $$= \frac{1}{s^2+1} \boxed{+ e^{-\pi s}\mathcal{L}\{\sin t\}}$$

4 $$= \boxed{\frac{1}{s^2+1} + e^{-\pi s}\frac{1}{s^2+1}}$$

$$= (1+e^{-\pi s})\frac{1}{s^2+1}$$

$$\mathcal{L}\{f\} = \frac{\mathcal{L}\{f_T\}}{1-e^{-\pi s}}$$

4 $$= \boxed{\frac{1+e^{-\pi s}}{1-e^{-\pi s}} \; \frac{1}{s^2+1}}$$

IF $0 \le t < \pi$,

$$y = \boxed{-\frac{1}{2}\sin t + \frac{3}{4}e^{t} + \frac{21}{4}e^{-t}}$$ 3

IF $\pi \le t < 2\pi$,

$$y = -\frac{1}{2}\sin t + \frac{3}{4}e^{t} + \frac{21}{4}e^{-t} + 2\left(\frac{1}{2}\sin t + \frac{1}{4}e^{t-\pi} - \frac{1}{4}e^{\pi-t}\right)$$

$$= \boxed{\frac{1}{2}\sin t + \left(\frac{3}{4} + \frac{1}{2e^{\pi}}\right)e^{t} + \left(\frac{21}{4} - \frac{e^{\pi}}{2}\right)e^{-t}}$$ 4 BONUS

IF $2\pi \le t < 3\pi$,

$$y = \frac{1}{2}\sin t + \left(\frac{3}{4} + \frac{1}{2e^{\pi}}\right)e^{t} + \left(\frac{21}{4} - \frac{e^{\pi}}{2}\right)e^{-t}$$

$$+ 2\left(-\frac{1}{2}\sin t + \frac{1}{4}e^{t-2\pi} - \frac{1}{4}e^{2\pi-t}\right)$$

$$= \boxed{-\frac{1}{2}\sin t + \left(\frac{3}{4} + \frac{1}{2e^{\pi}} + \frac{1}{2e^{2\pi}}\right)e^{t} + \left(\frac{21}{4} - \frac{e^{\pi}}{2} - \frac{e^{2\pi}}{2}\right)e^{-t}}$$

6 BONUS

$$\frac{6s-5}{(s-1)(s+1)} = \boxed{\frac{\overset{\frac{1}{2}}{\cancel{A}}}{s-1} + \frac{\overset{\frac{11}{2}}{\cancel{B}}}{s+1}} \longrightarrow \boxed{6s-5 = A(s+1)+B(s-1)}$$

2

2

$s=1$: $\quad 1 = 2A \rightarrow A = \frac{1}{2}$

$s=-1$: $\quad -11 = -2B \rightarrow B = \frac{11}{2}$

SANITY CHECK: $s=2$

2

$$\boxed{\begin{aligned} &\text{LHS} \quad \frac{12-5}{1(3)} = \frac{7}{3} \\ &\text{RHS} \quad \frac{\frac{1}{2}}{1} + \frac{\frac{11}{2}}{3} \\ &= \frac{1}{2} + \frac{11}{6} \\ &= \frac{3+11}{6} = \frac{14}{6} = \frac{7}{3} \end{aligned}}$$

$$\frac{1}{(s^2+1)(s+1)(s-1)} = \boxed{\frac{\overset{0}{\cancel{A}}s+\overset{-\frac{1}{2}}{\cancel{B}}}{s^2+1} + \frac{\overset{-\frac{1}{4}}{\cancel{C}}}{s+1} + \frac{\overset{\frac{1}{4}}{\cancel{D}}}{s-1}}$$

4

4

$$\boxed{1 = As(s+1)(s-1) + B(s+1)(s-1) + C(s^2+1)(s-1) + D(s^2+1)(s+1)}$$

$s=1$: $\quad 1 = D(2)(2) \longrightarrow D = \frac{1}{4}$

$s=-1$: $\quad 1 = C(2)(-2) \longrightarrow C = -\frac{1}{4}$

$s=0$: $\quad 1 = B(1)(-1) + C(1)(-1) + D(1)(1) \longrightarrow B = -C+D-1 = -\frac{1}{2}$

COEF OF s^3: $\quad 0 = A+C+D \longrightarrow A = -C-D \longrightarrow A = 0$

SANITY CHECK $s=2$

4

$$\boxed{\begin{aligned} &\text{LHS} \quad \frac{1}{5(3)(1)} = \frac{1}{15} \\ &\text{RHS} \quad \frac{-\frac{1}{2}}{5} - \frac{\frac{1}{4}}{3} + \frac{\frac{1}{4}}{1} \\ &= -\frac{1}{10} - \frac{1}{12} + \frac{1}{4} \\ &= \frac{-6-5+15}{60} \\ &= \frac{4}{60} = \frac{1}{15} \end{aligned}}$$

$$\frac{1+e^{-\pi s}}{1-e^{-\pi s}} = \frac{1}{1-e^{-\pi s}} + e^{-\pi s}\,\frac{1}{1-e^{-\pi s}}$$

$$= \boxed{\begin{aligned}&\left(1 + e^{-\pi s} + e^{-2\pi s} + e^{-3\pi s} + \cdots\right)\\ &+ \left(e^{-\pi s} + e^{-2\pi s} + e^{-3\pi s} + \cdots\right)\end{aligned}}$$

6

$$= 1 + 2e^{-\pi s} + 2e^{-2\pi s} + 2e^{-3\pi s} + \cdots$$

$$= 1 + \sum_{n=1}^{\infty} 2e^{-n\pi s}$$